10.6.5 EXERCISES

For a link to all of the additional resources available for this section, click OSttS Chapter 10 materials.

In Exercises 1 - 40, find the exact value.

For help with these exercises, click the resource below:

- The Inverse Trigonometric (Circular) Functions

1. $\arcsin(-1)$
2. $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$
3. $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$
4. $\arcsin\left(-\frac{1}{2}\right)$
5. $\arcsin(0)$
6. $\arcsin\left(\frac{1}{2}\right)$
7. $\arcsin\left(\frac{\sqrt{2}}{2}\right)$
8. $\arcsin\left(\frac{\sqrt{3}}{2}\right)$
9. $\arcsin(1)$
10. $\arccos(-1)$
11. $\arccos\left(-\frac{\sqrt{3}}{2}\right)$
12. $\arccos\left(-\frac{\sqrt{2}}{2}\right)$
13. $\arccos\left(-\frac{1}{2}\right)$
14. $\arccos(0)$
15. $\arccos\left(\frac{1}{2}\right)$
16. $\arccos\left(\frac{\sqrt{2}}{2}\right)$
17. $\arccos\left(\frac{\sqrt{3}}{2}\right)$
18. $\arccos(1)$
19. $\arctan\left(-\sqrt{3}\right)$
20. $\arctan(-1)$
21. $\arctan\left(-\frac{\sqrt{3}}{3}\right)$
22. $\arctan(0)$
23. $\arctan\left(\frac{\sqrt{3}}{3}\right)$
24. $\arctan(1)$
25. $\arctan\left(\sqrt{3}\right)$
26. $\operatorname{arccot}\left(-\sqrt{3}\right)$
27. $\operatorname{arccot}(-1)$
28. $\operatorname{arccot}\left(-\frac{\sqrt{3}}{3}\right)$
29. $\operatorname{arccot}(0)$
30. $\operatorname{arccot}\left(\frac{\sqrt{3}}{3}\right)$
31. $\operatorname{arccot}(1)$
32. $\operatorname{arccot}\left(\sqrt{3}\right)$
33. $\operatorname{arcsec}(2)$
34. $\operatorname{arccsc}(2)$
35. $\operatorname{arcsec}\left(\sqrt{2}\right)$
36. $\operatorname{arccsc}\left(\sqrt{2}\right)$
37. $\operatorname{arcsec}\left(\frac{2\sqrt{3}}{3}\right)$
38. $\operatorname{arccsc}\left(\frac{2\sqrt{3}}{3}\right)$
39. $\operatorname{arcsec}(1)$
40. $\operatorname{arccsc}(1)$

In Exercises 41 - 48, assume that the range of arcsecant is $\left[0, \frac{\pi}{2}\right) \cup \left[\pi, \frac{3\pi}{2}\right)$ and that the range of arccosecant is $\left(0, \frac{\pi}{2}\right] \cup \left(\pi, \frac{3\pi}{2}\right]$ when finding the exact value.

41. $\operatorname{arcsec}(-2)$
42. $\operatorname{arcsec}\left(-\sqrt{2}\right)$
43. $\operatorname{arcsec}\left(-\frac{2\sqrt{3}}{3}\right)$
44. $\operatorname{arcsec}(-1)$
45. $\operatorname{arccsc}(-2)$
46. $\operatorname{arccsc}\left(-\sqrt{2}\right)$
47. $\operatorname{arccsc}\left(-\frac{2\sqrt{3}}{3}\right)$
48. $\operatorname{arccsc}(-1)$

In Exercises 49 - 56, assume that the range of arcsecant is $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$ and that the range of arccosecant is $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$ when finding the exact value.

49. $\operatorname{arcsec}(-2)$
50. $\operatorname{arcsec}\left(-\sqrt{2}\right)$
51. $\operatorname{arcsec}\left(-\frac{2\sqrt{3}}{3}\right)$
52. $\operatorname{arcsec}(-1)$
53. $\operatorname{arccsc}(-2)$
54. $\operatorname{arccsc}\left(-\sqrt{2}\right)$
55. $\operatorname{arccsc}\left(-\frac{2\sqrt{3}}{3}\right)$
56. $\operatorname{arccsc}(-1)$

In Exercises 57 - 86, find the exact value or state that it is undefined.

57. $\sin\left(\arcsin\left(\frac{1}{2}\right)\right)$
58. $\sin\left(\arcsin\left(-\frac{\sqrt{2}}{2}\right)\right)$
59. $\sin\left(\arcsin\left(\frac{3}{5}\right)\right)$
60. $\sin(\arcsin(-0.42))$
61. $\sin\left(\arcsin\left(\frac{5}{4}\right)\right)$
62. $\cos\left(\arccos\left(\frac{\sqrt{2}}{2}\right)\right)$
63. $\cos\left(\arccos\left(-\frac{1}{2}\right)\right)$
64. $\cos\left(\arccos\left(\frac{5}{13}\right)\right)$
65. $\cos(\arccos(-0.998))$
66. $\cos(\arccos(\pi))$
67. $\tan(\arctan(-1))$
68. $\tan\left(\arctan\left(\sqrt{3}\right)\right)$
69. $\tan\left(\arctan\left(\frac{5}{12}\right)\right)$
70. $\tan(\arctan(0.965))$
71. $\tan(\arctan(3\pi))$
72. $\cot(\operatorname{arccot}(1))$
73. $\cot\left(\operatorname{arccot}\left(-\sqrt{3}\right)\right)$
74. $\cot\left(\operatorname{arccot}\left(-\frac{7}{24}\right)\right)$
75. $\cot(\operatorname{arccot}(-0.001))$
76. $\cot\left(\operatorname{arccot}\left(\frac{17\pi}{4}\right)\right)$
77. $\sec(\operatorname{arcsec}(2))$
78. $\sec(\operatorname{arcsec}(-1))$
79. $\sec\left(\operatorname{arcsec}\left(\frac{1}{2}\right)\right)$
80. $\sec(\operatorname{arcsec}(0.75))$

81. $\sec\left(\operatorname{arcsec}\left(117\pi\right)\right)$
82. $\csc\left(\operatorname{arccsc}\left(\sqrt{2}\right)\right)$
83. $\csc\left(\operatorname{arccsc}\left(-\frac{2\sqrt{3}}{3}\right)\right)$
84. $\csc\left(\operatorname{arccsc}\left(\frac{\sqrt{2}}{2}\right)\right)$
85. $\csc\left(\operatorname{arccsc}\left(1.0001\right)\right)$
86. $\csc\left(\operatorname{arccsc}\left(\frac{\pi}{4}\right)\right)$

In Exercises 87 - 106, find the exact value or state that it is undefined.

87. $\arcsin\left(\sin\left(\frac{\pi}{6}\right)\right)$
88. $\arcsin\left(\sin\left(-\frac{\pi}{3}\right)\right)$
89. $\arcsin\left(\sin\left(\frac{3\pi}{4}\right)\right)$
90. $\arcsin\left(\sin\left(\frac{11\pi}{6}\right)\right)$
91. $\arcsin\left(\sin\left(\frac{4\pi}{3}\right)\right)$
92. $\arccos\left(\cos\left(\frac{\pi}{4}\right)\right)$
93. $\arccos\left(\cos\left(\frac{2\pi}{3}\right)\right)$
94. $\arccos\left(\cos\left(\frac{3\pi}{2}\right)\right)$
95. $\arccos\left(\cos\left(-\frac{\pi}{6}\right)\right)$
96. $\arccos\left(\cos\left(\frac{5\pi}{4}\right)\right)$
97. $\arctan\left(\tan\left(\frac{\pi}{3}\right)\right)$
98. $\arctan\left(\tan\left(-\frac{\pi}{4}\right)\right)$
99. $\arctan\left(\tan\left(\pi\right)\right)$
100. $\arctan\left(\tan\left(\frac{\pi}{2}\right)\right)$
101. $\arctan\left(\tan\left(\frac{2\pi}{3}\right)\right)$
102. $\operatorname{arccot}\left(\cot\left(\frac{\pi}{3}\right)\right)$
103. $\operatorname{arccot}\left(\cot\left(-\frac{\pi}{4}\right)\right)$
104. $\operatorname{arccot}\left(\cot\left(\pi\right)\right)$
105. $\operatorname{arccot}\left(\cot\left(\frac{\pi}{2}\right)\right)$
106. $\operatorname{arccot}\left(\cot\left(\frac{2\pi}{3}\right)\right)$

In Exercises 107 - 118, assume that the range of arcsecant is $\left[0, \frac{\pi}{2}\right) \cup \left[\pi, \frac{3\pi}{2}\right)$ and that the range of arccosecant is $\left(0, \frac{\pi}{2}\right] \cup \left(\pi, \frac{3\pi}{2}\right]$ when finding the exact value.

107. $\operatorname{arcsec}\left(\sec\left(\frac{\pi}{4}\right)\right)$
108. $\operatorname{arcsec}\left(\sec\left(\frac{4\pi}{3}\right)\right)$
109. $\operatorname{arcsec}\left(\sec\left(\frac{5\pi}{6}\right)\right)$
110. $\operatorname{arcsec}\left(\sec\left(-\frac{\pi}{2}\right)\right)$
111. $\operatorname{arcsec}\left(\sec\left(\frac{5\pi}{3}\right)\right)$
112. $\operatorname{arccsc}\left(\csc\left(\frac{\pi}{6}\right)\right)$
113. $\operatorname{arccsc}\left(\csc\left(\frac{5\pi}{4}\right)\right)$
114. $\operatorname{arccsc}\left(\csc\left(\frac{2\pi}{3}\right)\right)$
115. $\operatorname{arccsc}\left(\csc\left(-\frac{\pi}{2}\right)\right)$
116. $\operatorname{arccsc}\left(\csc\left(\frac{11\pi}{6}\right)\right)$
117. $\operatorname{arcsec}\left(\sec\left(\frac{11\pi}{12}\right)\right)$
118. $\operatorname{arccsc}\left(\csc\left(\frac{9\pi}{8}\right)\right)$

In Exercises 119 - 130, assume that the range of arcsecant is $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$ and that the range of arccosecant is $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$ when finding the exact value.

119. $\operatorname{arcsec}\left(\sec\left(\frac{\pi}{4}\right)\right)$
120. $\operatorname{arcsec}\left(\sec\left(\frac{4\pi}{3}\right)\right)$
121. $\operatorname{arcsec}\left(\sec\left(\frac{5\pi}{6}\right)\right)$
122. $\operatorname{arcsec}\left(\sec\left(-\frac{\pi}{2}\right)\right)$
123. $\operatorname{arcsec}\left(\sec\left(\frac{5\pi}{3}\right)\right)$
124. $\operatorname{arccsc}\left(\csc\left(\frac{\pi}{6}\right)\right)$
125. $\operatorname{arccsc}\left(\csc\left(\frac{5\pi}{4}\right)\right)$
126. $\operatorname{arccsc}\left(\csc\left(\frac{2\pi}{3}\right)\right)$
127. $\operatorname{arccsc}\left(\csc\left(-\frac{\pi}{2}\right)\right)$
128. $\operatorname{arccsc}\left(\csc\left(\frac{11\pi}{6}\right)\right)$
129. $\operatorname{arcsec}\left(\sec\left(\frac{11\pi}{12}\right)\right)$
130. $\operatorname{arccsc}\left(\csc\left(\frac{9\pi}{8}\right)\right)$

In Exercises 131 - 154, find the exact value or state that it is undefined.
For help with these exercises, click one or more of the resources below:

- The Inverse Trigonometric (Circular) Functions
- Using the Quotient, Reciprocal, and Pythagorean Identities

131. $\sin\left(\arccos\left(-\frac{1}{2}\right)\right)$
132. $\sin\left(\arccos\left(\frac{3}{5}\right)\right)$
133. $\sin\left(\arctan\left(-2\right)\right)$
134. $\sin\left(\operatorname{arccot}\left(\sqrt{5}\right)\right)$
135. $\sin\left(\operatorname{arccsc}\left(-3\right)\right)$
136. $\cos\left(\arcsin\left(-\frac{5}{13}\right)\right)$
137. $\cos\left(\arctan\left(\sqrt{7}\right)\right)$
138. $\cos\left(\operatorname{arccot}\left(3\right)\right)$
139. $\cos\left(\operatorname{arcsec}\left(5\right)\right)$
140. $\tan\left(\arcsin\left(-\frac{2\sqrt{5}}{5}\right)\right)$
141. $\tan\left(\arccos\left(-\frac{1}{2}\right)\right)$
142. $\tan\left(\operatorname{arcsec}\left(\frac{5}{3}\right)\right)$
143. $\tan\left(\operatorname{arccot}\left(12\right)\right)$
144. $\cot\left(\arcsin\left(\frac{12}{13}\right)\right)$
145. $\cot\left(\arccos\left(\frac{\sqrt{3}}{2}\right)\right)$
146. $\cot\left(\operatorname{arccsc}\left(\sqrt{5}\right)\right)$
147. $\cot\left(\arctan\left(0.25\right)\right)$
148. $\sec\left(\arccos\left(\frac{\sqrt{3}}{2}\right)\right)$
149. $\sec\left(\arcsin\left(-\frac{12}{13}\right)\right)$
150. $\sec\left(\arctan\left(10\right)\right)$
151. $\sec\left(\operatorname{arccot}\left(-\frac{\sqrt{10}}{10}\right)\right)$
152. $\csc\left(\operatorname{arccot}\left(9\right)\right)$
153. $\csc\left(\arcsin\left(\frac{3}{5}\right)\right)$
154. $\csc\left(\arctan\left(-\frac{2}{3}\right)\right)$

In Exercises 155 - 164, find the exact value or state that it is undefined.
For help with these exercises, click one or more of the resources below:

- The Inverse Trigonometric (Circular) Functions
- Using the Quotient, Reciprocal, and Pythagorean Identities
- Using the Double Angle Identities
- Using the Half Angle Identities

155. $\sin\left(\arcsin\left(\frac{5}{13}\right)+\frac{\pi}{4}\right)$

156. $\cos\left(\operatorname{arcsec}(3)+\arctan(2)\right)$

157. $\tan\left(\arctan(3)+\arccos\left(-\frac{3}{5}\right)\right)$

158. $\sin\left(2\arcsin\left(-\frac{4}{5}\right)\right)$

159. $\sin\left(2\operatorname{arccsc}\left(\frac{13}{5}\right)\right)$

160. $\sin\left(2\arctan\left(2\right)\right)$

161. $\cos\left(2\arcsin\left(\frac{3}{5}\right)\right)$

162. $\cos\left(2\operatorname{arcsec}\left(\frac{25}{7}\right)\right)$

163. $\cos\left(2\operatorname{arccot}\left(-\sqrt{5}\right)\right)$

164. $\sin\left(\frac{\arctan(2)}{2}\right)$

In Exercises 165 - 184, rewrite the quantity as algebraic expressions of x and state the domain on which the equivalence is valid.

165. $\sin\left(\arccos\left(x\right)\right)$

166. $\cos\left(\arctan\left(x\right)\right)$

167. $\tan\left(\arcsin\left(x\right)\right)$

168. $\sec\left(\arctan\left(x\right)\right)$

169. $\csc\left(\arccos\left(x\right)\right)$

170. $\sin\left(2\arctan\left(x\right)\right)$

171. $\sin\left(2\arccos\left(x\right)\right)$

172. $\cos\left(2\arctan\left(x\right)\right)$

173. $\sin(\arccos(2x))$

174. $\sin\left(\arccos\left(\frac{x}{5}\right)\right)$

175. $\cos\left(\arcsin\left(\frac{x}{2}\right)\right)$

176. $\cos\left(\arctan\left(3x\right)\right)$

177. $\sin(2\arcsin(7x))$

178. $\sin\left(2\arcsin\left(\frac{x\sqrt{3}}{3}\right)\right)$

179. $\cos(2\arcsin(4x))$

180. $\sec(\arctan(2x))\tan(\arctan(2x))$

181. $\sin\left(\arcsin(x)+\arccos(x)\right)$

182. $\cos\left(\arcsin(x)+\arctan(x)\right)$

183. $\tan(2\arcsin(x))$

184. $\sin\left(\frac{1}{2}\arctan(x)\right)$

185. If $\sin(\theta) = \frac{x}{2}$ for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, find an expression for $\theta + \sin(2\theta)$ in terms of x.

186. If $\tan(\theta) = \frac{x}{7}$ for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, find an expression for $\frac{1}{2}\theta - \frac{1}{2}\sin(2\theta)$ in terms of x.

187. If $\sec(\theta) = \frac{x}{4}$ for $0 < \theta < \frac{\pi}{2}$, find an expression for $4\tan(\theta) - 4\theta$ in terms of x.

In Exercises 188 - 207, solve the equation using the techniques discussed in Example 10.6.7 then approximate the solutions which lie in the interval $[0, 2\pi)$ to four decimal places.

188. $\sin(x) = \frac{7}{11}$

189. $\cos(x) = -\frac{2}{9}$

190. $\sin(x) = -0.569$

191. $\cos(x) = 0.117$

192. $\sin(x) = 0.008$

193. $\cos(x) = \frac{359}{360}$

194. $\tan(x) = 117$

195. $\cot(x) = -12$

196. $\sec(x) = \frac{3}{2}$

197. $\csc(x) = -\frac{90}{17}$

198. $\tan(x) = -\sqrt{10}$

199. $\sin(x) = \frac{3}{8}$

200. $\cos(x) = -\frac{7}{16}$

201. $\tan(x) = 0.03$

202. $\sin(x) = 0.3502$

203. $\sin(x) = -0.721$

204. $\cos(x) = 0.9824$

205. $\cos(x) = -0.5637$

206. $\cot(x) = \frac{1}{117}$

207. $\tan(x) = -0.6109$

In Exercises 208 - 210, find the two acute angles in the right triangle whose sides have the given lengths. Express your answers using degree measure rounded to two decimal places.

208. 3, 4 and 5

209. 5, 12 and 13

210. 336, 527 and 625

For help with Exercises 211 - 215, click one or more of the resources below:

- Solving right triangles
- Solving application problems with right triangle trigonometry

211. A guy wire 1000 feet long is attached to the top of a tower. When pulled taut it touches level ground 360 feet from the base of the tower. What angle does the wire make with the ground? Express your answer using degree measure rounded to one decimal place.

212. At Cliffs of Insanity Point, The Great Sasquatch Canyon is 7117 feet deep. From that point, a fire is seen at a location known to be 10 miles away from the base of the sheer canyon wall. What angle of depression is made by the line of sight from the canyon edge to the fire? Express your answer using degree measure rounded to one decimal place.

213. Shelving is being built at the Utility Muffin Research Library which is to be 14 inches deep. An 18-inch rod will be attached to the wall and the underside of the shelf at its edge away from the wall, forming a right triangle under the shelf to support it. What angle, to the nearest degree, will the rod make with the wall?

214. A parasailor is being pulled by a boat on Lake Ippizuti. The cable is 300 feet long and the parasailor is 100 feet above the surface of the water. What is the angle of elevation from the boat to the parasailor? Express your answer using degree measure rounded to one decimal place.

215. A tag-and-release program to study the Sasquatch population of the eponymous Sasquatch National Park is begun. From a 200 foot tall tower, a ranger spots a Sasquatch lumbering through the wilderness directly towards the tower. Let θ denote the angle of depression from the top of the tower to a point on the ground. If the range of the rifle with a tranquilizer dart is 300 feet, find the smallest value of θ for which the corresponding point on the ground is in range of the rifle. Round your answer to the nearest hundreth of a degree.

In Exercises 216 - 221, rewrite the given function as a sinusoid of the form $S(x) = A\sin(\omega x + \phi)$ using Exercises 35 and 36 in Section 10.5 for reference. Approximate the value of ϕ (which is in radians, of course) to four decimal places.

216. $f(x) = 5\sin(3x) + 12\cos(3x)$

217. $f(x) = 3\cos(2x) + 4\sin(2x)$

218. $f(x) = \cos(x) - 3\sin(x)$

219. $f(x) = 7\sin(10x) - 24\cos(10x)$

220. $f(x) = -\cos(x) - 2\sqrt{2}\sin(x)$

221. $f(x) = 2\sin(x) - \cos(x)$

In Exercises 222 - 233, find the domain of the given function. Write your answers in interval notation.

222. $f(x) = \arcsin(5x)$

223. $f(x) = \arccos\left(\dfrac{3x-1}{2}\right)$

224. $f(x) = \arcsin\left(2x^2\right)$

225. $f(x) = \arccos\left(\dfrac{1}{x^2-4}\right)$

226. $f(x) = \arctan(4x)$

227. $f(x) = \operatorname{arccot}\left(\dfrac{2x}{x^2-9}\right)$

228. $f(x) = \arctan(\ln(2x-1))$

229. $f(x) = \operatorname{arccot}(\sqrt{2x-1})$

230. $f(x) = \operatorname{arcsec}(12x)$

231. $f(x) = \text{arccsc}(x+5)$ 232. $f(x) = \text{arcsec}\left(\frac{x^3}{8}\right)$ 233. $f(x) = \text{arccsc}\left(e^{2x}\right)$

234. Show that $\text{arcsec}(x) = \arccos\left(\frac{1}{x}\right)$ for $|x| \geq 1$ as long as we use $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$ as the range of $f(x) = \text{arcsec}(x)$.

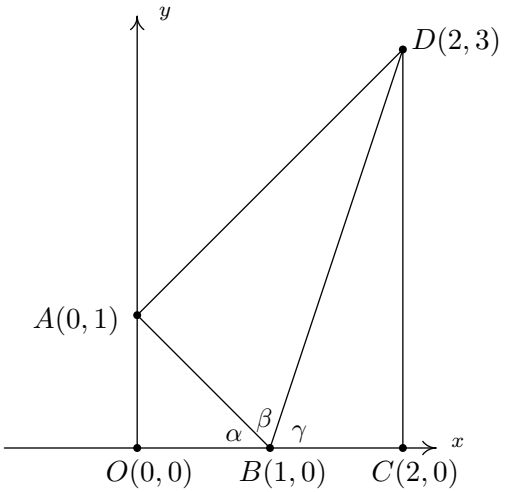
235. Show that $\text{arccsc}(x) = \arcsin\left(\frac{1}{x}\right)$ for $|x| \geq 1$ as long as we use $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$ as the range of $f(x) = \text{arccsc}(x)$.

236. Show that $\arcsin(x) + \arccos(x) = \frac{\pi}{2}$ for $-1 \leq x \leq 1$.

237. Discuss with your classmates why $\arcsin\left(\frac{1}{2}\right) \neq 30°$.

238. Use the following picture and the series of exercises on the next page to show that

$$\arctan(1) + \arctan(2) + \arctan(3) = \pi$$



(a) Clearly $\triangle AOB$ and $\triangle BCD$ are right triangles because the line through O and A and the line through C and D are perpendicular to the x-axis. Use the distance formula to show that $\triangle BAD$ is also a right triangle (with $\angle BAD$ being the right angle) by showing that the sides of the triangle satisfy the Pythagorean Theorem.

(b) Use $\triangle AOB$ to show that $\alpha = \arctan(1)$

(c) Use $\triangle BAD$ to show that $\beta = \arctan(2)$

(d) Use $\triangle BCD$ to show that $\gamma = \arctan(3)$

(e) Use the fact that O, B and C all lie on the x-axis to conclude that $\alpha + \beta + \gamma = \pi$. Thus $\arctan(1) + \arctan(2) + \arctan(3) = \pi$.

Checkpoint Quiz 10.6

1. Simplify:
   (a) $\arcsin\left(\sin\left(\frac{11\pi}{12}\right)\right)$
   (b) $\sin\left(\arcsin(0.3)\right)$
   (c) $\cos\left(\arcsin(0.3)\right)$
2. Simplify: $\sin\left(\operatorname{arcsec}(5) + \arctan(-2)\right)$
3. Simplify: $\sin\left(2\arcsin(3x)\right)$
4. Solve for the following equations:
   (a) $\sin(\theta) = \frac{3}{5}$
   (b) $\cos(t) = -0.33$
   (c) $\tan(x) = -5$
5. Write $f(x) = \cos(7x) - 2\sin(7x)$ in the form $S(x) = A\sin(\omega x + \phi)$.

For worked out solutions to this quiz, click the links below:

- Quiz Solution Part 1
- Quiz Solution Part 2
- Quiz Solution Part 3
- Quiz Solution Part 4

10.6.6 Answers

1. $\arcsin(-1) = -\frac{\pi}{2}$
2. $\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$
3. $\arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$
4. $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$
5. $\arcsin(0) = 0$
6. $\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$
7. $\arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$
8. $\arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$
9. $\arcsin(1) = \frac{\pi}{2}$
10. $\arccos(-1) = \pi$
11. $\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$
12. $\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$
13. $\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$
14. $\arccos(0) = \frac{\pi}{2}$
15. $\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$
16. $\arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$
17. $\arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$
18. $\arccos(1) = 0$
19. $\arctan\left(-\sqrt{3}\right) = -\frac{\pi}{3}$
20. $\arctan(-1) = -\frac{\pi}{4}$
21. $\arctan\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$
22. $\arctan(0) = 0$
23. $\arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$
24. $\arctan(1) = \frac{\pi}{4}$
25. $\arctan\left(\sqrt{3}\right) = \frac{\pi}{3}$
26. $\operatorname{arccot}\left(-\sqrt{3}\right) = \frac{5\pi}{6}$
27. $\operatorname{arccot}(-1) = \frac{3\pi}{4}$
28. $\operatorname{arccot}\left(-\frac{\sqrt{3}}{3}\right) = \frac{2\pi}{3}$
29. $\operatorname{arccot}(0) = \frac{\pi}{2}$
30. $\operatorname{arccot}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{3}$
31. $\operatorname{arccot}(1) = \frac{\pi}{4}$
32. $\operatorname{arccot}\left(\sqrt{3}\right) = \frac{\pi}{6}$
33. $\operatorname{arcsec}(2) = \frac{\pi}{3}$
34. $\operatorname{arccsc}(2) = \frac{\pi}{6}$
35. $\operatorname{arcsec}\left(\sqrt{2}\right) = \frac{\pi}{4}$
36. $\operatorname{arccsc}\left(\sqrt{2}\right) = \frac{\pi}{4}$
37. $\operatorname{arcsec}\left(\frac{2\sqrt{3}}{3}\right) = \frac{\pi}{6}$
38. $\operatorname{arccsc}\left(\frac{2\sqrt{3}}{3}\right) = \frac{\pi}{3}$
39. $\operatorname{arcsec}(1) = 0$
40. $\operatorname{arccsc}(1) = \frac{\pi}{2}$
41. $\operatorname{arcsec}(-2) = \frac{4\pi}{3}$
42. $\operatorname{arcsec}\left(-\sqrt{2}\right) = \frac{5\pi}{4}$

43. $\operatorname{arcsec}\left(-\frac{2\sqrt{3}}{3}\right) = \frac{7\pi}{6}$
44. $\operatorname{arcsec}(-1) = \pi$
45. $\operatorname{arccsc}(-2) = \frac{7\pi}{6}$
46. $\operatorname{arccsc}\left(-\sqrt{2}\right) = \frac{5\pi}{4}$
47. $\operatorname{arccsc}\left(-\frac{2\sqrt{3}}{3}\right) = \frac{4\pi}{3}$
48. $\operatorname{arccsc}(-1) = \frac{3\pi}{2}$
49. $\operatorname{arcsec}(-2) = \frac{2\pi}{3}$
50. $\operatorname{arcsec}\left(-\sqrt{2}\right) = \frac{3\pi}{4}$
51. $\operatorname{arcsec}\left(-\frac{2\sqrt{3}}{3}\right) = \frac{5\pi}{6}$
52. $\operatorname{arcsec}(-1) = \pi$
53. $\operatorname{arccsc}(-2) = -\frac{\pi}{6}$
54. $\operatorname{arccsc}\left(-\sqrt{2}\right) = -\frac{\pi}{4}$
55. $\operatorname{arccsc}\left(-\frac{2\sqrt{3}}{3}\right) = -\frac{\pi}{3}$
56. $\operatorname{arccsc}(-1) = -\frac{\pi}{2}$
57. $\sin\left(\arcsin\left(\frac{1}{2}\right)\right) = \frac{1}{2}$
58. $\sin\left(\arcsin\left(-\frac{\sqrt{2}}{2}\right)\right) = -\frac{\sqrt{2}}{2}$
59. $\sin\left(\arcsin\left(\frac{3}{5}\right)\right) = \frac{3}{5}$
60. $\sin(\arcsin(-0.42)) = -0.42$
61. $\sin\left(\arcsin\left(\frac{5}{4}\right)\right)$ is undefined.
62. $\cos\left(\arccos\left(\frac{\sqrt{2}}{2}\right)\right) = \frac{\sqrt{2}}{2}$
63. $\cos\left(\arccos\left(-\frac{1}{2}\right)\right) = -\frac{1}{2}$
64. $\cos\left(\arccos\left(\frac{5}{13}\right)\right) = \frac{5}{13}$
65. $\cos(\arccos(-0.998)) = -0.998$
66. $\cos(\arccos(\pi))$ is undefined.
67. $\tan(\arctan(-1)) = -1$
68. $\tan\left(\arctan\left(\sqrt{3}\right)\right) = \sqrt{3}$
69. $\tan\left(\arctan\left(\frac{5}{12}\right)\right) = \frac{5}{12}$
70. $\tan(\arctan(0.965)) = 0.965$
71. $\tan(\arctan(3\pi)) = 3\pi$
72. $\cot(\operatorname{arccot}(1)) = 1$
73. $\cot\left(\operatorname{arccot}\left(-\sqrt{3}\right)\right) = -\sqrt{3}$
74. $\cot\left(\operatorname{arccot}\left(-\frac{7}{24}\right)\right) = -\frac{7}{24}$
75. $\cot(\operatorname{arccot}(-0.001)) = -0.001$
76. $\cot\left(\operatorname{arccot}\left(\frac{17\pi}{4}\right)\right) = \frac{17\pi}{4}$
77. $\sec(\operatorname{arcsec}(2)) = 2$
78. $\sec(\operatorname{arcsec}(-1)) = -1$

79. $\sec\left(\text{arcsec}\left(\frac{1}{2}\right)\right)$ is undefined.

80. $\sec\left(\text{arcsec}\left(0.75\right)\right)$ is undefined.

81. $\sec\left(\text{arcsec}\left(\frac{\pi}{2}\right)\right) = \frac{\pi}{2}$

82. $\csc\left(\text{arccsc}\left(\sqrt{2}\right)\right) = \sqrt{2}$

83. $\csc\left(\text{arccsc}\left(-\frac{2\sqrt{3}}{3}\right)\right) = -\frac{2\sqrt{3}}{3}$

84. $\csc\left(\text{arccsc}\left(\frac{\sqrt{2}}{2}\right)\right)$ is undefined.

85. $\csc\left(\text{arccsc}\left(1.0001\right)\right) = 1.0001$

86. $\csc\left(\text{arccsc}\left(\frac{\pi}{4}\right)\right)$ is undefined.

87. $\arcsin\left(\sin\left(\frac{\pi}{6}\right)\right) = \frac{\pi}{6}$

88. $\arcsin\left(\sin\left(-\frac{\pi}{3}\right)\right) = -\frac{\pi}{3}$

89. $\arcsin\left(\sin\left(\frac{3\pi}{4}\right)\right) = \frac{\pi}{4}$

90. $\arcsin\left(\sin\left(\frac{11\pi}{6}\right)\right) = -\frac{\pi}{6}$

91. $\arcsin\left(\sin\left(\frac{4\pi}{3}\right)\right) = -\frac{\pi}{3}$

92. $\arccos\left(\cos\left(\frac{\pi}{4}\right)\right) = \frac{\pi}{4}$

93. $\arccos\left(\cos\left(\frac{2\pi}{3}\right)\right) = \frac{2\pi}{3}$

94. $\arccos\left(\cos\left(\frac{3\pi}{2}\right)\right) = \frac{\pi}{2}$

95. $\arccos\left(\cos\left(-\frac{\pi}{6}\right)\right) = \frac{\pi}{6}$

96. $\arccos\left(\cos\left(\frac{5\pi}{4}\right)\right) = \frac{3\pi}{4}$

97. $\arctan\left(\tan\left(\frac{\pi}{3}\right)\right) = \frac{\pi}{3}$

98. $\arctan\left(\tan\left(-\frac{\pi}{4}\right)\right) = -\frac{\pi}{4}$

99. $\arctan\left(\tan\left(\pi\right)\right) = 0$

100. $\arctan\left(\tan\left(\frac{\pi}{2}\right)\right)$ is undefined

101. $\arctan\left(\tan\left(\frac{2\pi}{3}\right)\right) = -\frac{\pi}{3}$

102. $\text{arccot}\left(\cot\left(\frac{\pi}{3}\right)\right) = \frac{\pi}{3}$

103. $\text{arccot}\left(\cot\left(-\frac{\pi}{4}\right)\right) = \frac{3\pi}{4}$

104. $\text{arccot}\left(\cot\left(\pi\right)\right)$ is undefined

105. $\text{arccot}\left(\cot\left(\frac{3\pi}{2}\right)\right) = \frac{\pi}{2}$

106. $\text{arccot}\left(\cot\left(\frac{2\pi}{3}\right)\right) = \frac{2\pi}{3}$

107. $\text{arcsec}\left(\sec\left(\frac{\pi}{4}\right)\right) = \frac{\pi}{4}$

108. $\text{arcsec}\left(\sec\left(\frac{4\pi}{3}\right)\right) = \frac{4\pi}{3}$

109. $\text{arcsec}\left(\sec\left(\frac{5\pi}{6}\right)\right) = \frac{7\pi}{6}$

110. $\text{arcsec}\left(\sec\left(-\frac{\pi}{2}\right)\right)$ is undefined.

111. $\text{arcsec}\left(\sec\left(\frac{5\pi}{3}\right)\right) = \frac{\pi}{3}$

112. $\text{arccsc}\left(\csc\left(\frac{\pi}{6}\right)\right) = \frac{\pi}{6}$

113. $\operatorname{arccsc}\left(\csc\left(\frac{5\pi}{4}\right)\right) = \frac{5\pi}{4}$

114. $\operatorname{arccsc}\left(\csc\left(\frac{2\pi}{3}\right)\right) = \frac{\pi}{3}$

115. $\operatorname{arccsc}\left(\csc\left(-\frac{\pi}{2}\right)\right) = \frac{3\pi}{2}$

116. $\operatorname{arccsc}\left(\csc\left(\frac{11\pi}{6}\right)\right) = \frac{7\pi}{6}$

117. $\operatorname{arcsec}\left(\sec\left(\frac{11\pi}{12}\right)\right) = \frac{13\pi}{12}$

118. $\operatorname{arccsc}\left(\csc\left(\frac{9\pi}{8}\right)\right) = \frac{9\pi}{8}$

119. $\operatorname{arcsec}\left(\sec\left(\frac{\pi}{4}\right)\right) = \frac{\pi}{4}$

120. $\operatorname{arcsec}\left(\sec\left(\frac{4\pi}{3}\right)\right) = \frac{2\pi}{3}$

121. $\operatorname{arcsec}\left(\sec\left(\frac{5\pi}{6}\right)\right) = \frac{5\pi}{6}$

122. $\operatorname{arcsec}\left(\sec\left(-\frac{\pi}{2}\right)\right)$ is undefined.

123. $\operatorname{arcsec}\left(\sec\left(\frac{5\pi}{3}\right)\right) = \frac{\pi}{3}$

124. $\operatorname{arccsc}\left(\csc\left(\frac{\pi}{6}\right)\right) = \frac{\pi}{6}$

125. $\operatorname{arccsc}\left(\csc\left(\frac{5\pi}{4}\right)\right) = -\frac{\pi}{4}$

126. $\operatorname{arccsc}\left(\csc\left(\frac{2\pi}{3}\right)\right) = \frac{\pi}{3}$

127. $\operatorname{arccsc}\left(\csc\left(-\frac{\pi}{2}\right)\right) = -\frac{\pi}{2}$

128. $\operatorname{arccsc}\left(\csc\left(\frac{11\pi}{6}\right)\right) = -\frac{\pi}{6}$

129. $\operatorname{arcsec}\left(\sec\left(\frac{11\pi}{12}\right)\right) = \frac{11\pi}{12}$

130. $\operatorname{arccsc}\left(\csc\left(\frac{9\pi}{8}\right)\right) = -\frac{\pi}{8}$

131. $\sin\left(\arccos\left(-\frac{1}{2}\right)\right) = \frac{\sqrt{3}}{2}$

132. $\sin\left(\arccos\left(\frac{3}{5}\right)\right) = \frac{4}{5}$

133. $\sin\left(\arctan\left(-2\right)\right) = -\frac{2\sqrt{5}}{5}$

134. $\sin\left(\operatorname{arccot}\left(\sqrt{5}\right)\right) = \frac{\sqrt{6}}{6}$

135. $\sin\left(\operatorname{arccsc}\left(-3\right)\right) = -\frac{1}{3}$

136. $\cos\left(\arcsin\left(-\frac{5}{13}\right)\right) = \frac{12}{13}$

137. $\cos\left(\arctan\left(\sqrt{7}\right)\right) = \frac{\sqrt{2}}{4}$

138. $\cos\left(\operatorname{arccot}\left(3\right)\right) = \frac{3\sqrt{10}}{10}$

139. $\cos\left(\operatorname{arcsec}\left(5\right)\right) = \frac{1}{5}$

140. $\tan\left(\arcsin\left(-\frac{2\sqrt{5}}{5}\right)\right) = -2$

141. $\tan\left(\arccos\left(-\frac{1}{2}\right)\right) = -\sqrt{3}$

142. $\tan\left(\operatorname{arcsec}\left(\frac{5}{3}\right)\right) = \frac{4}{3}$

143. $\tan\left(\operatorname{arccot}\left(12\right)\right) = \frac{1}{12}$

144. $\cot\left(\arcsin\left(\frac{12}{13}\right)\right) = \frac{5}{12}$

145. $\cot\left(\arccos\left(\frac{\sqrt{3}}{2}\right)\right) = \sqrt{3}$

146. $\cot\left(\operatorname{arccsc}\left(\sqrt{5}\right)\right) = 2$

147. $\cot\left(\arctan\left(0.25\right)\right) = 4$

148. $\sec\left(\arccos\left(\frac{\sqrt{3}}{2}\right)\right) = \frac{2\sqrt{3}}{3}$

149. $\sec\left(\arcsin\left(-\frac{12}{13}\right)\right) = \frac{13}{5}$

150. $\sec\left(\arctan\left(10\right)\right) = \sqrt{101}$

151. $\sec\left(\operatorname{arccot}\left(-\frac{\sqrt{10}}{10}\right)\right) = -\sqrt{11}$

152. $\csc\left(\operatorname{arccot}\left(9\right)\right) = \sqrt{82}$

153. $\csc\left(\arcsin\left(\frac{3}{5}\right)\right) = \frac{5}{3}$

154. $\csc\left(\arctan\left(-\frac{2}{3}\right)\right) = -\frac{\sqrt{13}}{2}$

155. $\sin\left(\arcsin\left(\frac{5}{13}\right) + \frac{\pi}{4}\right) = \frac{17\sqrt{2}}{26}$

156. $\cos\left(\operatorname{arcsec}(3) + \arctan(2)\right) = \frac{\sqrt{5} - 4\sqrt{10}}{15}$

157. $\tan\left(\arctan(3) + \arccos\left(-\frac{3}{5}\right)\right) = \frac{1}{3}$

158. $\sin\left(2\arcsin\left(-\frac{4}{5}\right)\right) = -\frac{24}{25}$

159. $\sin\left(2\operatorname{arccsc}\left(\frac{13}{5}\right)\right) = \frac{120}{169}$

160. $\sin\left(2\arctan\left(2\right)\right) = \frac{4}{5}$

161. $\cos\left(2\arcsin\left(\frac{3}{5}\right)\right) = \frac{7}{25}$

162. $\cos\left(2\operatorname{arcsec}\left(\frac{25}{7}\right)\right) = -\frac{527}{625}$

163. $\cos\left(2\operatorname{arccot}\left(-\sqrt{5}\right)\right) = \frac{2}{3}$

164. $\sin\left(\frac{\arctan(2)}{2}\right) = \sqrt{\frac{5-\sqrt{5}}{10}}$

165. $\sin\left(\arccos\left(x\right)\right) = \sqrt{1-x^2}$ for $-1 \leq x \leq 1$

166. $\cos\left(\arctan\left(x\right)\right) = \frac{1}{\sqrt{1+x^2}}$ for all x

167. $\tan\left(\arcsin\left(x\right)\right) = \frac{x}{\sqrt{1-x^2}}$ for $-1 < x < 1$

168. $\sec\left(\arctan\left(x\right)\right) = \sqrt{1+x^2}$ for all x

169. $\csc\left(\arccos\left(x\right)\right) = \frac{1}{\sqrt{1-x^2}}$ for $-1 < x < 1$

170. $\sin\left(2\arctan\left(x\right)\right) = \frac{2x}{x^2+1}$ for all x

171. $\sin\left(2\arccos\left(x\right)\right) = 2x\sqrt{1-x^2}$ for $-1 \leq x \leq 1$

172. $\cos(2\arctan(x)) = \dfrac{1-x^2}{1+x^2}$ for all x

173. $\sin(\arccos(2x)) = \sqrt{1-4x^2}$ for $-\frac{1}{2} \le x \le \frac{1}{2}$

174. $\sin\left(\arccos\left(\dfrac{x}{5}\right)\right) = \dfrac{\sqrt{25-x^2}}{5}$ for $-5 \le x \le 5$

175. $\cos\left(\arcsin\left(\dfrac{x}{2}\right)\right) = \dfrac{\sqrt{4-x^2}}{2}$ for $-2 \le x \le 2$

176. $\cos(\arctan(3x)) = \dfrac{1}{\sqrt{1+9x^2}}$ for all x

177. $\sin(2\arcsin(7x)) = 14x\sqrt{1-49x^2}$ for $-\dfrac{1}{7} \le x \le \dfrac{1}{7}$

178. $\sin\left(2\arcsin\left(\dfrac{x\sqrt{3}}{3}\right)\right) = \dfrac{2x\sqrt{3-x^2}}{3}$ for $-\sqrt{3} \le x \le \sqrt{3}$

179. $\cos(2\arcsin(4x)) = 1 - 32x^2$ for $-\dfrac{1}{4} \le x \le \dfrac{1}{4}$

180. $\sec(\arctan(2x))\tan(\arctan(2x)) = 2x\sqrt{1+4x^2}$ for all x

181. $\sin(\arcsin(x) + \arccos(x)) = 1$ for $-1 \le x \le 1$

182. $\cos(\arcsin(x) + \arctan(x)) = \dfrac{\sqrt{1-x^2} - x^2}{\sqrt{1+x^2}}$ for $-1 \le x \le 1$

183. [10] $\tan(2\arcsin(x)) = \dfrac{2x\sqrt{1-x^2}}{1-2x^2}$ for x in $\left(-1, -\dfrac{\sqrt{2}}{2}\right) \cup \left(-\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right) \cup \left(\dfrac{\sqrt{2}}{2}, 1\right)$

184. $\sin\left(\dfrac{1}{2}\arctan(x)\right) = \begin{cases} \sqrt{\dfrac{\sqrt{x^2+1}-1}{2\sqrt{x^2+1}}} & \text{for } x \ge 0 \\ -\sqrt{\dfrac{\sqrt{x^2+1}-1}{2\sqrt{x^2+1}}} & \text{for } x < 0 \end{cases}$

185. If $\sin(\theta) = \dfrac{x}{2}$ for $-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$, then $\theta + \sin(2\theta) = \arcsin\left(\dfrac{x}{2}\right) + \dfrac{x\sqrt{4-x^2}}{2}$

186. If $\tan(\theta) = \dfrac{x}{7}$ for $-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$, then $\dfrac{1}{2}\theta - \dfrac{1}{2}\sin(2\theta) = \dfrac{1}{2}\arctan\left(\dfrac{x}{7}\right) - \dfrac{7x}{x^2+49}$

[10] The equivalence for $x = \pm 1$ can be verified independently of the derivation of the formula, but Calculus is required to fully understand what is happening at those x values. You'll see what we mean when you work through the details of the identity for $\tan(2t)$. For now, we exclude $x = \pm 1$ from our answer.

187. If $\sec(\theta) = \frac{x}{4}$ for $0 < \theta < \frac{\pi}{2}$, then $4\tan(\theta) - 4\theta = \sqrt{x^2 - 16} - 4\text{arcsec}\left(\frac{x}{4}\right)$

188. $x = \arcsin\left(\frac{7}{11}\right) + 2\pi k$ or $x = \pi - \arcsin\left(\frac{7}{11}\right) + 2\pi k$, in $[0, 2\pi)$, $x \approx 0.6898, 2.4518$

189. $x = \arccos\left(-\frac{2}{9}\right) + 2\pi k$ or $x = -\arccos\left(-\frac{2}{9}\right) + 2\pi k$, in $[0, 2\pi)$, $x \approx 1.7949, 4.4883$

190. $x = \pi + \arcsin(0.569) + 2\pi k$ or $x = 2\pi - \arcsin(0.569) + 2\pi k$, in $[0, 2\pi)$, $x \approx 3.7469, 5.6779$

191. $x = \arccos(0.117) + 2\pi k$ or $x = 2\pi - \arccos(0.117) + 2\pi k$, in $[0, 2\pi)$, $x \approx 1.4535, 4.8297$

192. $x = \arcsin(0.008) + 2\pi k$ or $x = \pi - \arcsin(0.008) + 2\pi k$, in $[0, 2\pi)$, $x \approx 0.0080, 3.1336$

193. $x = \arccos\left(\frac{359}{360}\right) + 2\pi k$ or $x = 2\pi - \arccos\left(\frac{359}{360}\right) + 2\pi k$, in $[0, 2\pi)$, $x \approx 0.0746, 6.2086$

194. $x = \arctan(117) + \pi k$, in $[0, 2\pi)$, $x \approx 1.56225, 4.70384$

195. $x = \arctan\left(-\frac{1}{12}\right) + \pi k$, in $[0, 2\pi)$, $x \approx 3.0585, 6.2000$

196. $x = \arccos\left(\frac{2}{3}\right) + 2\pi k$ or $x = 2\pi - \arccos\left(\frac{2}{3}\right) + 2\pi k$, in $[0, 2\pi)$, $x \approx 0.8411, 5.4422$

197. $x = \pi + \arcsin\left(\frac{17}{90}\right) + 2\pi k$ or $x = 2\pi - \arcsin\left(\frac{17}{90}\right) + 2\pi k$, in $[0, 2\pi)$, $x \approx 3.3316, 6.0932$

198. $x = \arctan\left(-\sqrt{10}\right) + \pi k$, in $[0, 2\pi)$, $x \approx 1.8771, 5.0187$

199. $x = \arcsin\left(\frac{3}{8}\right) + 2\pi k$ or $x = \pi - \arcsin\left(\frac{3}{8}\right) + 2\pi k$, in $[0, 2\pi)$, $x \approx 0.3844, 2.7572$

200. $x = \arccos\left(-\frac{7}{16}\right) + 2\pi k$ or $x = -\arccos\left(-\frac{7}{16}\right) + 2\pi k$, in $[0, 2\pi)$, $x \approx 2.0236, 4.2596$

201. $x = \arctan(0.03) + \pi k$, in $[0, 2\pi)$, $x \approx 0.0300, 3.1716$

202. $x = \arcsin(0.3502) + 2\pi k$ or $x = \pi - \arcsin(0.3502) + 2\pi k$, in $[0, 2\pi)$, $x \approx 0.3578, 2.784$

203. $x = \pi + \arcsin(0.721) + 2\pi k$ or $x = 2\pi - \arcsin(0.721) + 2\pi k$, in $[0, 2\pi)$, $x \approx 3.9468, 5.4780$

204. $x = \arccos(0.9824) + 2\pi k$ or $x = 2\pi - \arccos(0.9824) + 2\pi k$, in $[0, 2\pi)$, $x \approx 0.1879, 6.0953$

205. $x = \arccos(-0.5637) + 2\pi k$ or $x = -\arccos(-0.5637) + 2\pi k$, in $[0, 2\pi)$, $x \approx 2.1697, 4.1135$

206. $x = \arctan(117) + \pi k$, in $[0, 2\pi)$, $x \approx 1.5622, 4.7038$

207. $x = \arctan(-0.6109) + \pi k$, in $[0, 2\pi)$, $x \approx 2.5932, 5.7348$

208. $36.87°$ and $53.13°$ 209. $22.62°$ and $67.38°$ 210. $32.52°$ and $57.48°$

211. $68.9°$ 212. $7.7°$ 213. $51°$ 214. $19.5°$ 215. $41.81°$

216. $f(x) = 5\sin(3x) + 12\cos(3x) = 13\sin\left(3x + \arcsin\left(\frac{12}{13}\right)\right) \approx 13\sin(3x + 1.1760)$

217. $f(x) = 3\cos(2x) + 4\sin(2x) = 5\sin\left(2x + \arcsin\left(\frac{3}{5}\right)\right) \approx 5\sin(2x + 0.6435)$

218. $f(x) = \cos(x) - 3\sin(x) = \sqrt{10}\sin\left(x + \arccos\left(-\frac{3\sqrt{10}}{10}\right)\right) \approx \sqrt{10}\sin(x + 2.8198)$

219. $f(x) = 7\sin(10x) - 24\cos(10x) = 25\sin\left(10x + \arcsin\left(-\frac{24}{25}\right)\right) \approx 25\sin(10x - 1.2870)$

220. $f(x) = -\cos(x) - 2\sqrt{2}\sin(x) = 3\sin\left(x + \pi + \arcsin\left(\frac{1}{3}\right)\right) \approx 3\sin(x + 3.4814)$

221. $f(x) = 2\sin(x) - \cos(x) = \sqrt{5}\sin\left(x + \arcsin\left(-\frac{\sqrt{5}}{5}\right)\right) \approx \sqrt{5}\sin(x - 0.4636)$

222. $\left[-\frac{1}{5}, \frac{1}{5}\right]$

223. $\left[-\frac{1}{3}, 1\right]$

224. $\left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$

225. $(-\infty, -\sqrt{5}] \cup [-\sqrt{3}, \sqrt{3}] \cup [\sqrt{5}, \infty)$

226. $(-\infty, \infty)$

227. $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

228. $\left(\frac{1}{2}, \infty\right)$

229. $\left[\frac{1}{2}, \infty\right)$

230. $\left(-\infty, -\frac{1}{12}\right] \cup \left[\frac{1}{12}, \infty\right)$

231. $(-\infty, -6] \cup [-4, \infty)$

232. $(-\infty, -2] \cup [2, \infty)$

233. $[0, \infty)$